

Smith-Purcell Free Electron Laser: A Compact Terhertz Source

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Indore**

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Argonne National Laboratory

In this talk, rather than making a review of Smith-Purcell free-electron lasers, I am going to share some of the things I learnt while working with Kwang-Je during 2003-05, and which continued

Outline of the talk

I. Motivation for Smith-Purcell (SP) FELs

II. Surface Mode in SP-FEL

III. Beam-wave interaction in SP-FEL

IV. Example cases of SP-FEL as terahertz source

V. Conclusions

Motivation for Smith-Purcell FELs

Coherent terahertz radiation can be produced using undulator based FELs.

But, we need **high energy electron accelerator** (~5-10 MeV), with radiation shielding.

Can we use **low energy electron beam** for terahertz generation?

Undulator based FEL with low energy?

$$\lambda_R = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \quad K = \frac{eB_u\lambda_u}{2\pi mc}$$

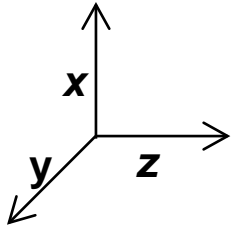
Need to use microundulators. Very low K . Gain will be low. FELs typically require high peak current, so bunched beam is needed.

Slow wave FEL with low energy?

Can we use slow wave structures, such as grating or dielectric based structures for this purpose?

If yes, why has it not been done so far. What are the challenges.

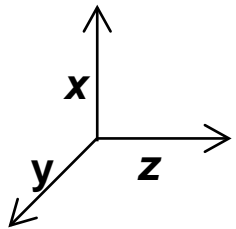
Fast and slow Waves



$$\begin{aligned} E_x &= E_0 e^{(ik_z z - \omega t)} \\ H_y &= H_0 e^{(ik_z z - \omega t)} \end{aligned}$$

$$v_{phase} = \frac{\omega}{k_z} = c$$

Allowed in free space (fast wave)



$$\begin{aligned} E_x &= E_0 e^{(ik_z z - \omega t)} e^{\pm \Gamma x} \\ H_y &= H_0 e^{(ik_z z - \omega t)} e^{\pm \Gamma x} \\ \mathbf{E_z} &= \mathbf{E_1} e^{(ik_z z - \omega t)} e^{\pm \Gamma x} \end{aligned}$$

$$v_{phase} = \frac{\omega}{k_z} < c$$

$$k_z^2 - \Gamma^2 = \frac{\omega^2}{c^2} \rightarrow \Gamma = \frac{2\pi}{\beta \gamma \lambda}$$

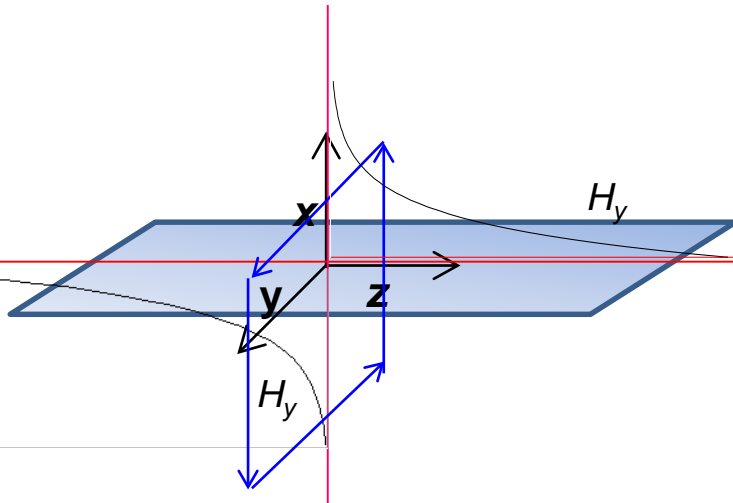
Allowed in free space, but needs some material to be present at the boundary to satisfy boundary conditions. (slow wave)

Energy propagated
in z-direction only,
not in x-direction

Slow waves generated by a modulated sheet electron beam

A modulated sheet electron beam moving with constant speed produces slow waves, decaying away in the transverse direction*

$$J_z = \delta(x) K_0(\omega) \exp[i(\alpha_0 z - \omega t)]$$



$$H_y^I = \frac{1}{2} \varepsilon(x) K_0(\omega) \exp[i\alpha_0 z - \varepsilon(x) \Gamma_0 x]$$

$$E_x^I = \frac{\alpha_0}{2\omega \varepsilon_0} \varepsilon(x) K_0(\omega) \exp[i\alpha_0 z - \varepsilon(x) \Gamma_0 x]$$

$$E_z^I = \frac{-i\Gamma_0}{2\omega \varepsilon_0} K_0(\omega) \exp[i\alpha_0 z - \varepsilon(x) \Gamma_0 x]$$

$$E_y^I = H_x^I = H_z^I = 0$$

$$\varepsilon(x) = -1 \text{ for } x < 0$$

$$+1 \text{ for } x > 0$$

$$\Gamma_0 = \sqrt{\alpha_0^2 - \omega^2 / c^2}$$

$$= \omega / c \beta \gamma = 2\pi / \beta \gamma \lambda$$

These are slow plane waves, propagating along z -axis with speed v , but decaying along x -axis with length scale of $\beta \gamma \lambda / 2\pi$. These are non-radiating, zeroth order evanescent wave.

* K.-J. Kim and S. -B. Song, NIMA 475 (2001) 158.

On the Theory of some Čerenkovian Effects (*).

G. TORALDO DI FRANCA

Istituto di Fisica della Radiazione - Università di Firenze

(ricevuto il 21 Dicembre 1959)



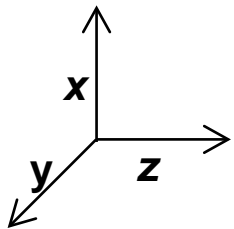
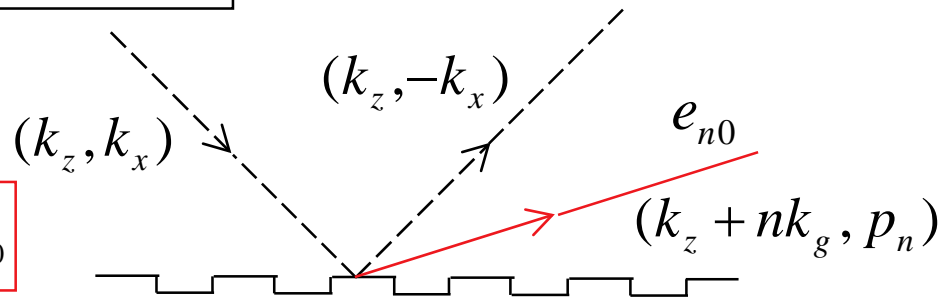
1916-2011

Summary. - The field generated by a charged particle in uniform straight motion is expanded into a set of evanescent waves. The expansion is valid in any half-space with no points in common with the path of the particle. The evanescent waves may impinge on the surface of an optical diffraction grating and be diffracted. Some of the diffracted waves turn out to be ordinary plane waves, which carry energy away from the grating. It is possible in this way to explain the Smith and Purcell effect and to derive some quantitative conclusions.

Reflection of evanescent wave by a metallic reflection grating

$$H_y = A_0^I \exp(i\alpha_0 z + \Gamma_0 x - i\omega t) + \sum_{n=-\infty}^{+\infty} A_n^R \exp(i\alpha_n z + ip_n x - i\omega t)$$

$$k_x = -i\Gamma_0, k_z = \alpha_0$$



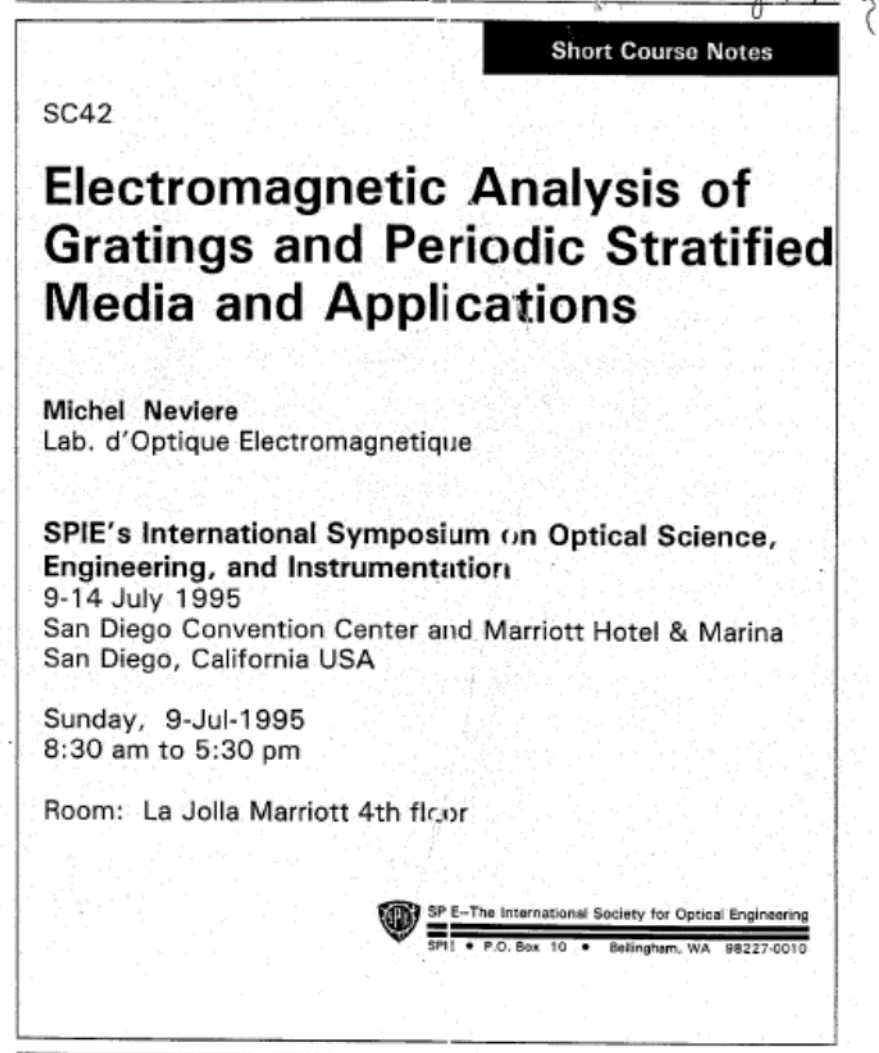
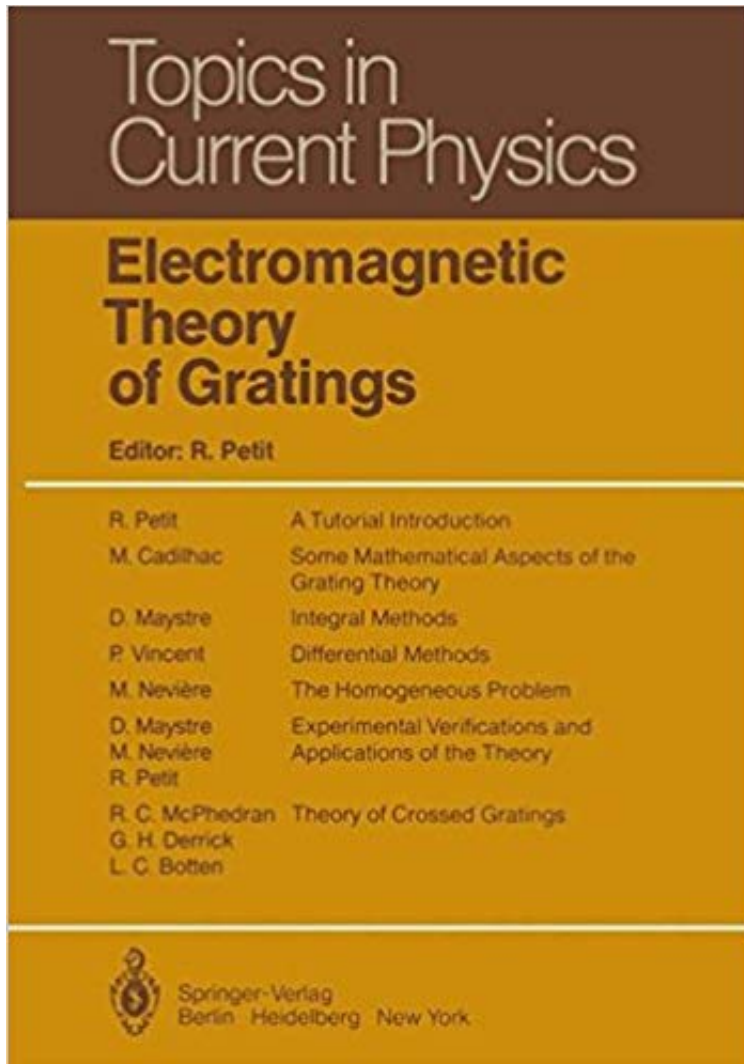
$$p_n = \sqrt{\frac{\omega^2}{c^2} - (k_z + nk_g)^2}$$

real → propagating, **radiative**

imaginary → evanescent, **non-radiative**

For some values of n (<0), k_x and k_z are real, those are SP radiation orders.

Diffraction at the grating surface



LETTERS TO THE EDITOR

1069

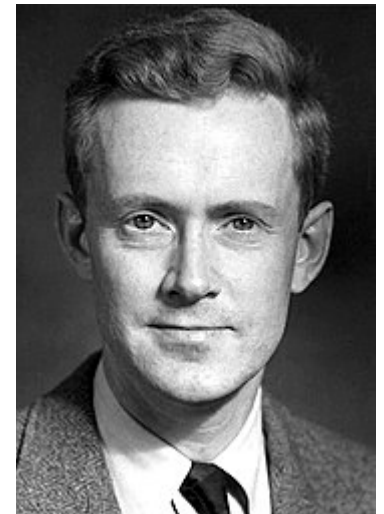
Visible Light from Localized Surface Charges Moving across a Grating

S. J. SMITH* AND E. M. PURCELL

Lyman Laboratory, Harvard University, Cambridge, Massachusetts

(Received September 25, 1953)

IT occurred to one of the authors (EMP) that if an electron passes close to the surface of a metal diffraction grating, moving at right angles to the rulings, the periodic motion of the charge induced on the surface of the grating should give rise to radiation. A simple Huygens construction shows the fundamental wavelength to be $d(\beta^{-1} - \cos\theta)$, in which d is the distance between rulings, β stands for v/c as usual, and θ is the angle between the direction of motion of the electron and the light ray. If $d = 1.67$ microns, as in a typical optical grating, and if electrons of energy around 300 kev are used, the light emitted forward should lie in the visible spectrum. As for intensity, if we assume that the surface charge traversing the hills and dales is equivalent to a point charge e oscillating with an amplitude $d/10$, we find that in the forward



1912-1997

Smith-Purcell radiation

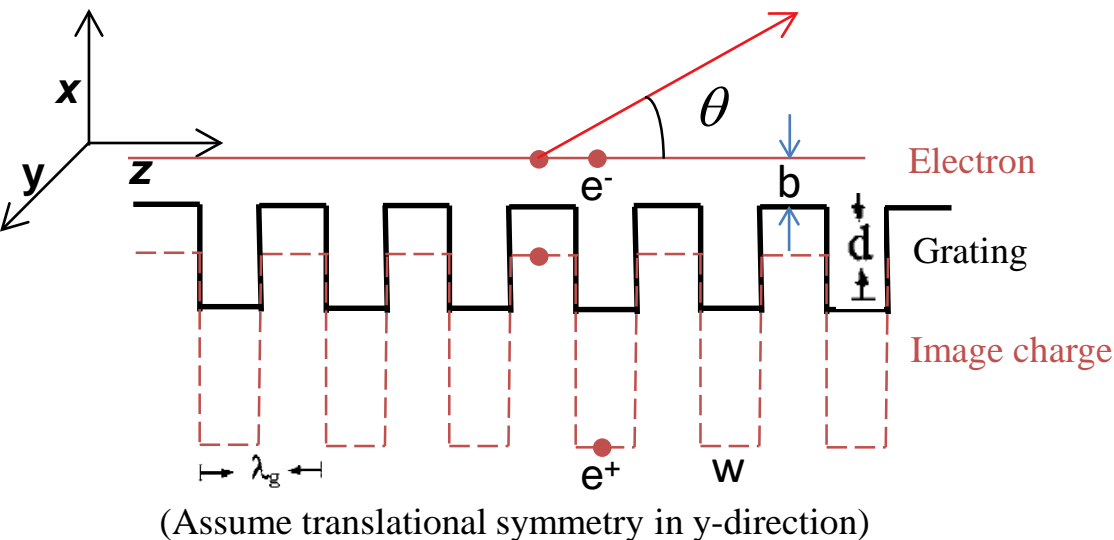
An electron beam traveling close and parallel to a metallic reflection grating, with grating rulings perpendicular to the electron motion gives off polarized electromagnetic radiation, known as SP radiation*.

Dartmouth parameters**

$$\lambda_g = 173 \mu\text{m}, \beta = 0.35 \text{ (35 keV)}$$

$$d = 100 \mu\text{m}, w = 62 \mu\text{m},$$

$$b = 10 \mu\text{m}, L = 12.7 \text{ mm}$$



$$\lambda = \frac{\lambda_g}{\beta} (1 - \beta \cos \theta)$$

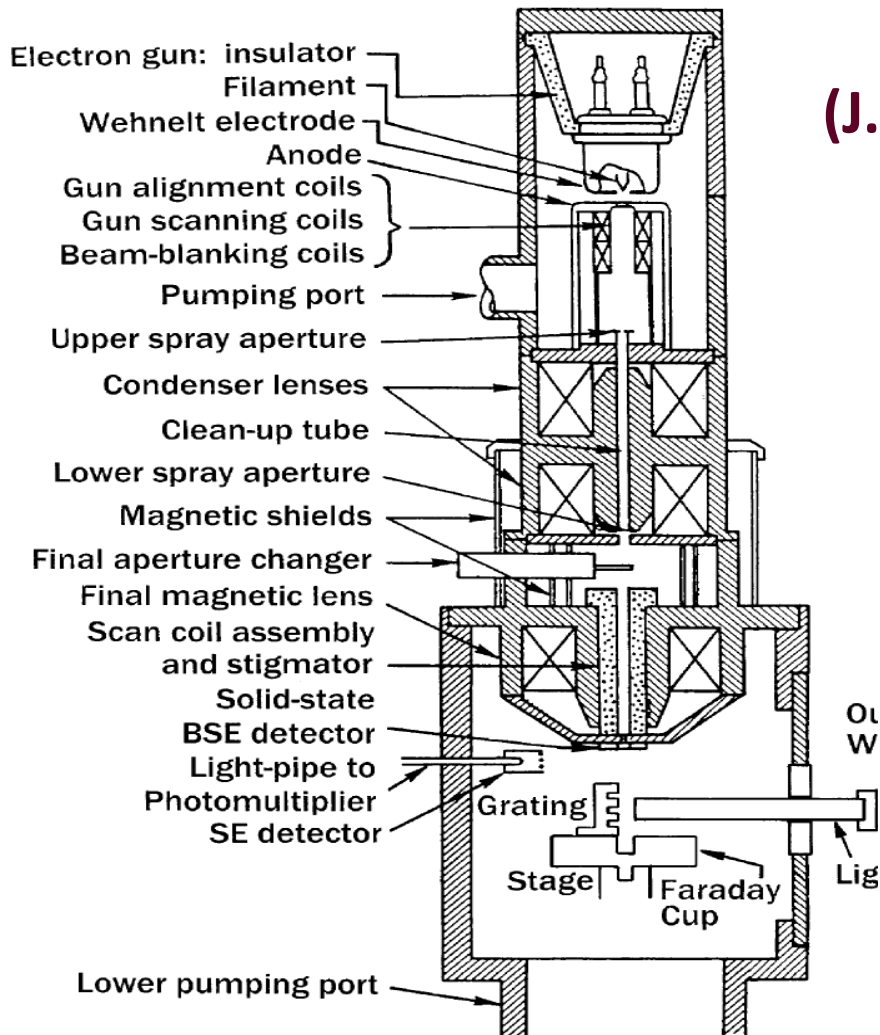
| θ | λ |
|----------|-------------------|
| 0 | 321 μm |
| $\pi/2$ | 494 μm |
| π | 667 μm |

$$\lambda_{\text{SP}} < 667 \mu\text{m}$$

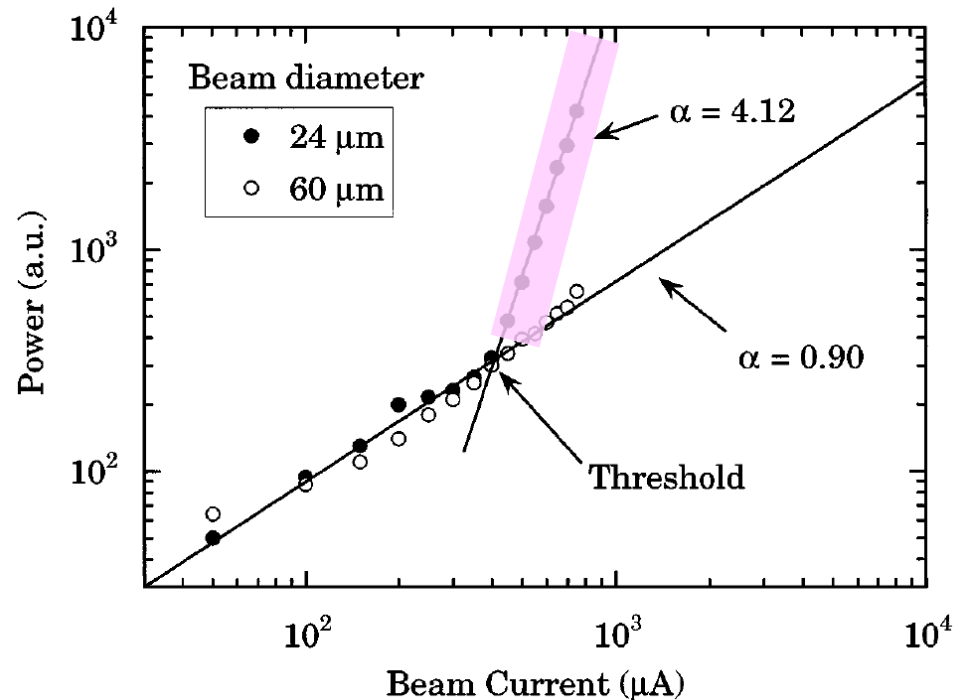
**J. Urata et al., Phys. Rev. Lett., 80, 516 (1998)

*S. J. Smith and E. M. Purcell, Phys. Rev. 92, 1069 (1953)

SEM-based Smith-Purcell FEL?

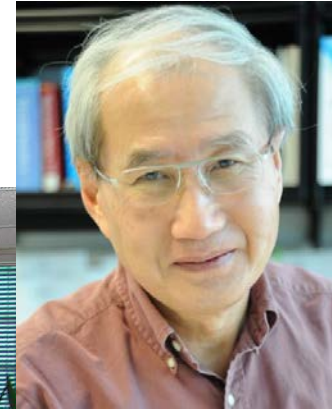


(J. Urata et al., PRL 80 (1998) 516-519)



Is it an FEL?

“Smith-Purcell FEL Lab” at the Univ. of Chicago following the Dartmouth set-up (O. Kapp, A. Crewe, Y.E. Sun, V. Kumar, KJK)



$\beta = 0.35$ (35 keV)
 $I = 1$ mA
 $\lambda_g = 173$ μm ,
 $d = 100$ mm,
 $w = 62$ μm ,
 $b = 10$ μm ,
 $L = 12.7$ mm

On the Theory of some Čerenkovian Effects (*).

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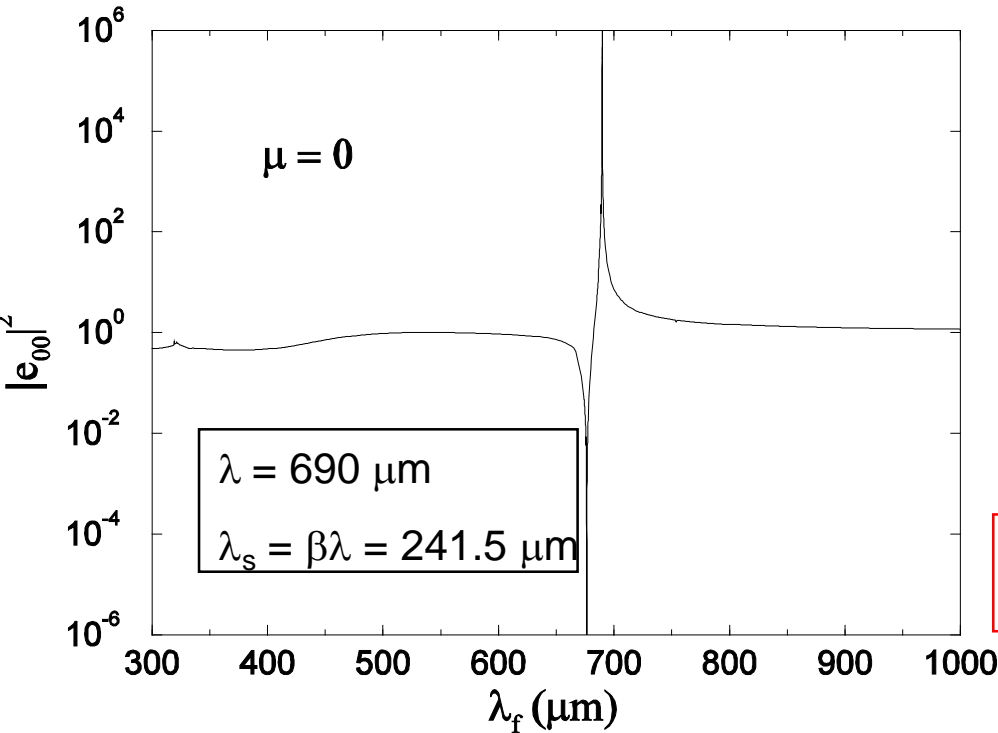
(ricevuto il 21 Dicembre 1959)

Summary. - The field generated by a charged particle in uniform straight motion is expanded into a set of evanescent waves. The expansion is valid in any half-space with no points in common with the path of the particle. The evanescent waves may impinge on the surface of an optical diffraction grating and be diffracted. Some of the diffracted waves turn out to be ordinary plane waves, which carry energy away from the grating. It is possible in this way to explain the Smith and Purcell effect and to derive some quantitative conclusions.

For instance, if we want to extract from the particle as much energy as possible, we are interested in having a great value for δ_m . It is interesting to note that when both the incident and reflected waves are evanescent, the grating can show a sort of resonance ⁽²¹⁾ giving $\delta_m \gg 1$. It will be worthwhile to investigate whether a similar condition can be obtained when the incident wave is evanescent and the diffracted wave is an ordinary travelling wave.

Singularity in reflectivity and surface mode in SP-FEL

*V. Kumar and K.-J. Kim, PAC05 proceedings, Phys Rev. E 73, 026501(2006)



Dartmouth parameters**

$\lambda_g = 173 \text{ } \mu\text{m}$, $\beta = 0.35$ (35 keV)

groove depth (d) = $100 \text{ } \mu\text{m}$,

groove width (w) = $62 \text{ } \mu\text{m}$,

$$H_y^{SM} = \sum C_n \exp(i\alpha_n z - \Gamma_n x - i\omega_s t)$$

- e_0 blows up at $\lambda = 690 \text{ } \mu\text{m}$ which means that the zeroth order outgoing wave is supported there even in the absence of incoming wave.

- In fact, zeroth order wave and all higher order outgoing components add up with specific amplitude ratio to satisfy the boundary condition. This is the surface mode supported self-consistently by the grating

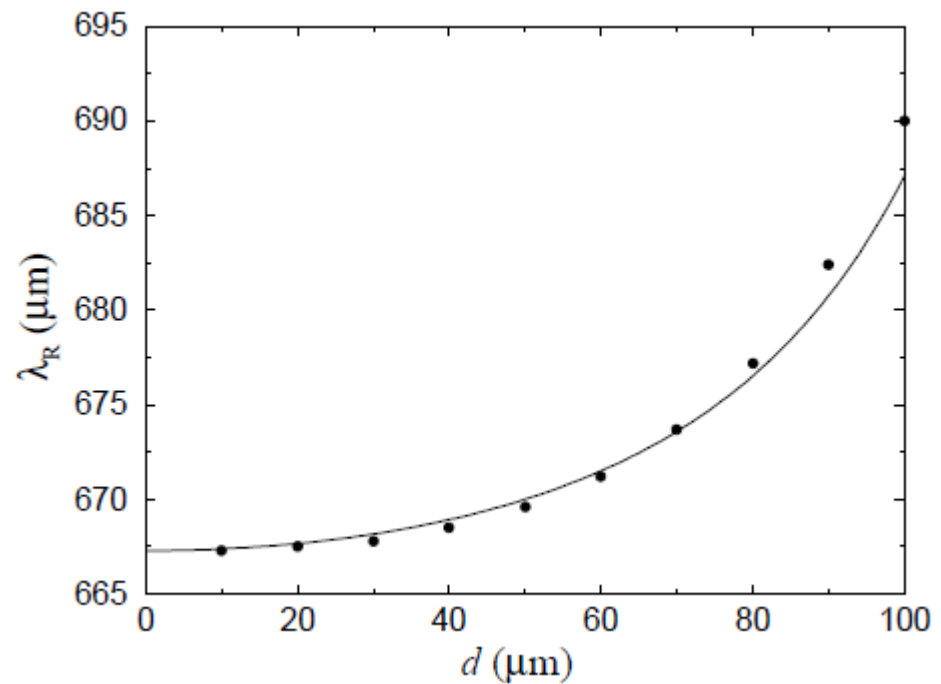
Approximate formula for resonant wavelength in SP-FEL

$$\lambda_R = \lambda_R^0 + \Delta\lambda$$

$$\lambda_R^0 = \lambda_g \frac{1 + \beta}{\beta}$$

$$\Delta\lambda = \frac{\lambda_g}{2} \left(\frac{\delta}{k_R^0} \right)^2$$

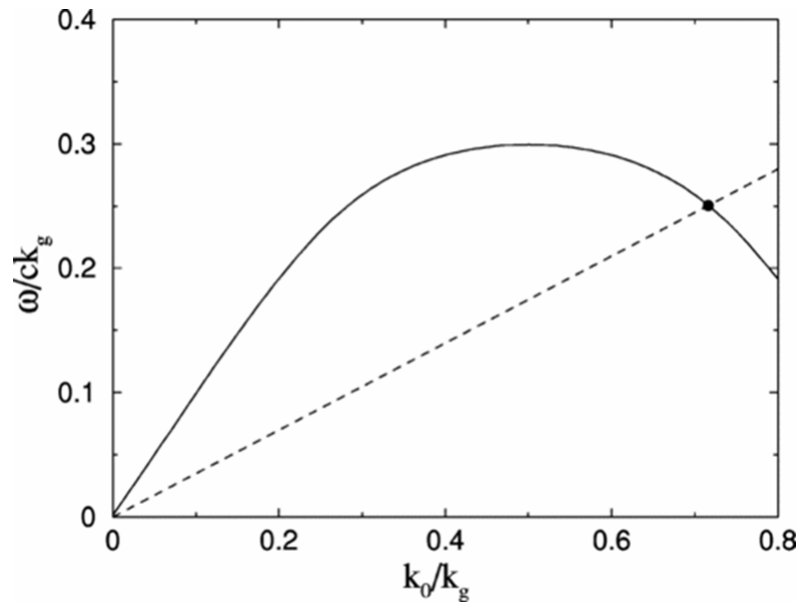
$$\delta = \frac{2 \tan(k_R^0 d)}{k_R^0 w \lambda_g} \{1 - \cos(w k_R^0)\}$$



Notice the difference: U-FEL: $\lambda = \lambda_u \frac{1-\beta}{\beta}$, SP-FEL: $\lambda \cong \lambda_g \frac{1+\beta}{\beta}$

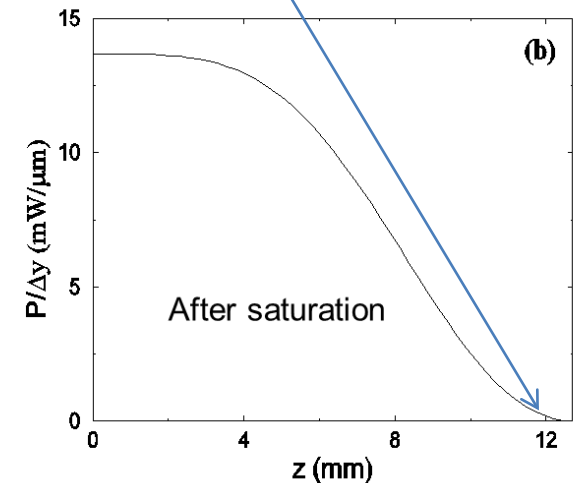
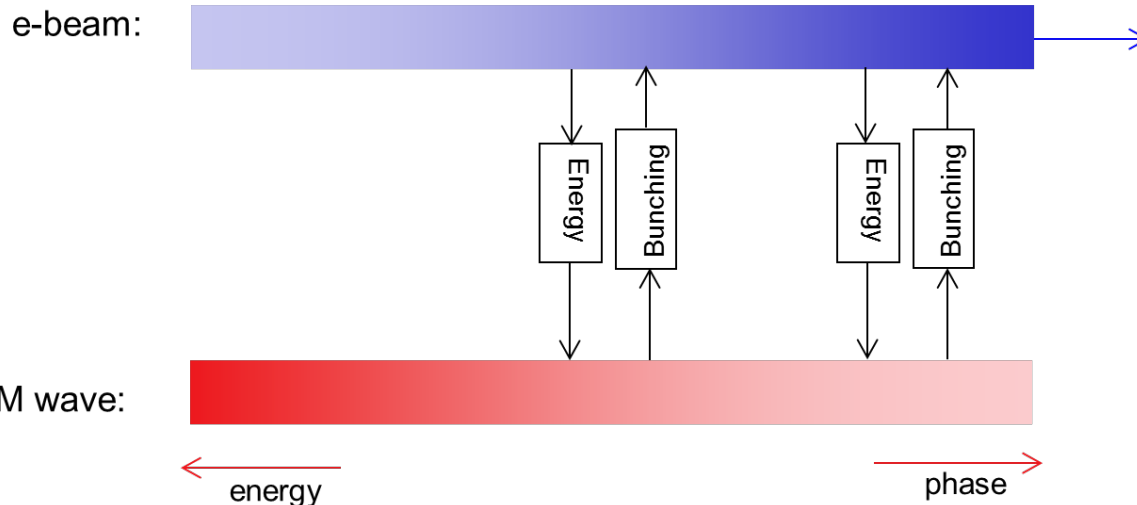
Beam-wave interaction

Interaction between slow wave and the co-propagating electron beam produces coherent radiation



Group velocity is typically -ve, works as a **backward wave oscillator**.

Boundary condition: Input field needs to be provided at the electron beam exit!



Singularity of reflection coefficient

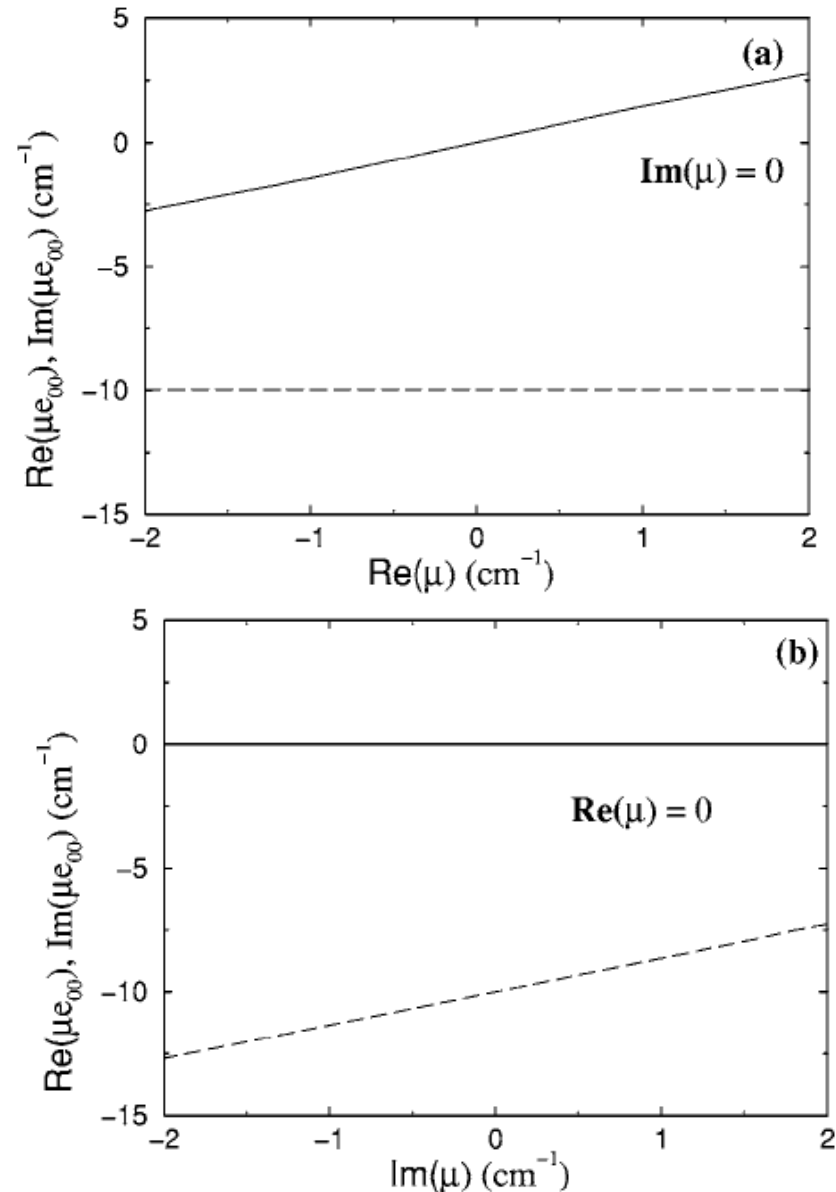
- We observe that the singularity in R is removed at non-zero growth rate (μ) of the incident wave.

$$E_Z = E_0 e^{(ik_z z - \omega t)} e^{\Gamma x} e^{\mu z}$$

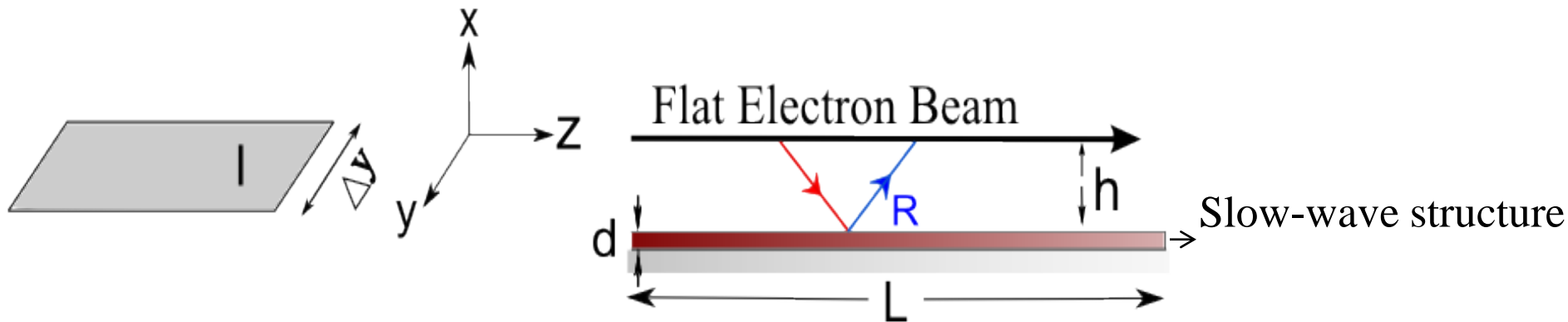
- R has a simple pole at $\mu = 0$, and it can be expressed as

$$R(\mu) = \frac{-i\chi}{\mu} + \chi_1$$

$$\chi = 10 \text{ per cm}, \chi_1 = 1.37$$



Maxwell-Lorentz equations - I



Surface current: $(I / \Delta y) \langle e^{-i\psi} \rangle e^{i(k_s z - \omega_s t)} + c.c.$

Axial electric field: $E_z(z, t) e^{i(k_s z - \omega_s t)} + c.c.$

$$E_z = \frac{iZ_0}{2\beta\gamma\Delta y} \left(R e^{-2\Gamma_0 b} - 1 \right) \langle e^{-i\psi} \rangle$$

$$E_z = \frac{iZ_0}{2\beta\gamma\Delta y} \left(\frac{-i\chi}{\mu} e^{-2\Gamma_0 b} + \chi_1 e^{-2\Gamma_0 b} - 1 \right) \langle e^{-i\psi} \rangle = E^{sur} + E^{sc}$$

A novel approach!

This approach of separating the total electromagnetic field into surface mode field and space charge field is similar to the approach described in Ref. [40], where it is stated that “*The total fields from an arbitrary, spatially periodic current are shown to consist of a pole term, which is identified as the structure field, and a remainder, which is identified as the space charge field*”. Dynamics of the electron beam is governed by the surface mode field, as well as the

[40]. R. J. Pierce, *Traveling Wave Tubes* (D. Van Nostrand Company Inc; First edition, 1950)

Maxwell-Lorentz equations - II

$$\mu E^{sur} = \frac{IZ_0 \chi}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b} \langle e^{-i\psi} \rangle$$

Replacing μ with the operator d/dz , we get

$$\frac{dE^{sur}}{dz} = \frac{IZ_0 \chi}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b} \langle e^{-i\psi} \rangle$$

Time-dependent Maxwell equation:

$$\begin{aligned} \frac{\partial E}{\partial t} - v_g \frac{\partial E}{\partial z} &= -\frac{IZ_0 \chi v_g}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b} \langle e^{-i\psi} \rangle \\ E^{sc} &= \frac{-iIZ_0}{2\beta\gamma\Delta y} (1 - \chi_1 e^{-2\Gamma_0 b}) \langle e^{-i\psi} \rangle \end{aligned}$$

Lorentz equation:

$$\begin{aligned} \frac{\partial \gamma_i}{\partial t} + v \frac{\partial \gamma_i}{\partial z} &= \frac{ev}{mc^2} (E + E^{sc}) e^{i\psi_i} + c.c. \\ \frac{\partial \psi_i}{\partial t} + v \frac{\partial \psi_i}{\partial z} &= \frac{\omega_s}{\beta^2 \gamma^2} \frac{(\gamma_i - \gamma_p)}{\gamma_p} \end{aligned}$$

No attenuation No end reflection No 3-D effects

Maxwell-Lorentz equations with attenuation and end reflections

Maxwell-Lorentz equations with attenuation and end reflections

Dimensionless variables:

$$\zeta = z / L$$

$$\tau = \left(\tau - \frac{z}{v_p} \right) \left(\frac{1}{v_p} + \frac{1}{v_g} \right)^{-1} \frac{1}{L}$$

$$\eta_i = \frac{k_s L}{\beta_p^3 \gamma_p^3} (\gamma_i - \gamma_p)$$

$$\varepsilon = \frac{e}{mc^2} \frac{k_s L^2}{c \beta_p^3 \gamma_p^3} E$$

$$\varepsilon_s = \frac{e}{mc^2} \frac{k_s L^2}{c \beta_p^3 \gamma_p^3} E_s$$

$$J = 2\pi \frac{I}{I_A} \frac{\chi}{\Delta y} \frac{k_s L^3}{\beta_p^4 \gamma_p^4} e^{-2\Gamma_0 b}$$

Maxwell-Lorentz equations in dimensionless variables:

$$\frac{\partial \varepsilon_-}{\partial \tau} - \frac{\partial \varepsilon_-}{\partial \zeta} = -J \langle e^{-i\psi} \rangle - \alpha L \varepsilon_-$$

$$\frac{\partial \varepsilon_+}{\partial \tau} + d_1 \frac{\partial \varepsilon_+}{\partial \zeta} = -\alpha L \varepsilon_+$$

$$\frac{\partial \eta_i}{\partial \zeta} = (\varepsilon_+ + \varepsilon^{sc}) e^{i\psi_i} + c.c.$$

$$\frac{\partial \psi_i}{\partial \zeta} = \eta_i$$

$$\varepsilon^{sc} = i \frac{J}{\chi L} (\chi_1 - e^{2\Gamma_0 b}) \langle e^{-i\psi} \rangle$$

Boundary conditions:

$$\varepsilon_+(\zeta = 1, \tau) = -\varepsilon_-(\zeta = 1, \tau),$$

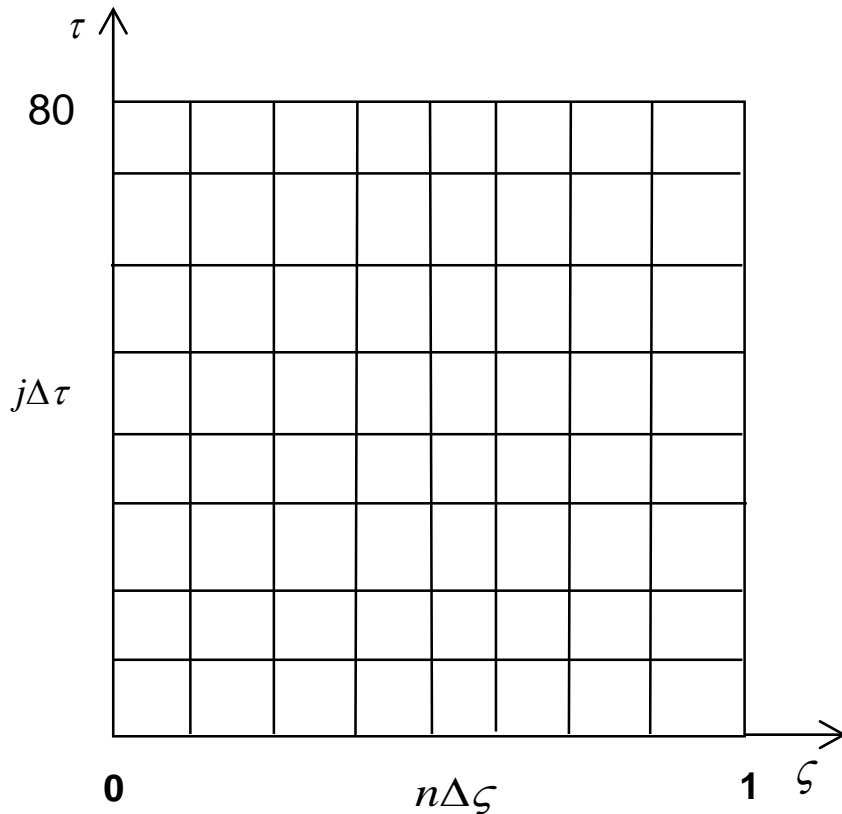
$$\varepsilon_-(\zeta = 0, \tau) = Re^{-i2k_0 L} \varepsilon_+(\zeta = 0, \tau)$$

ε_+ : forward wave

ε_- : backward wave

$$d_1 = \frac{v_p + v_g}{v_p - v_g}$$

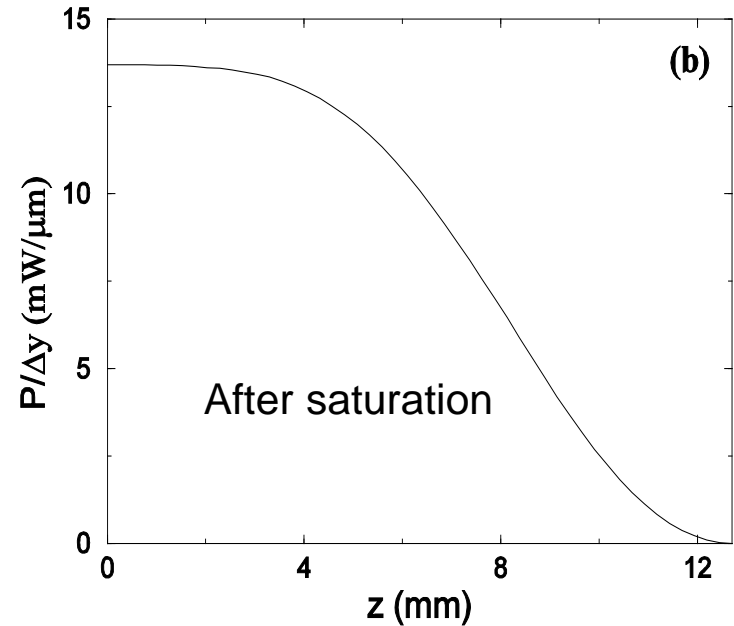
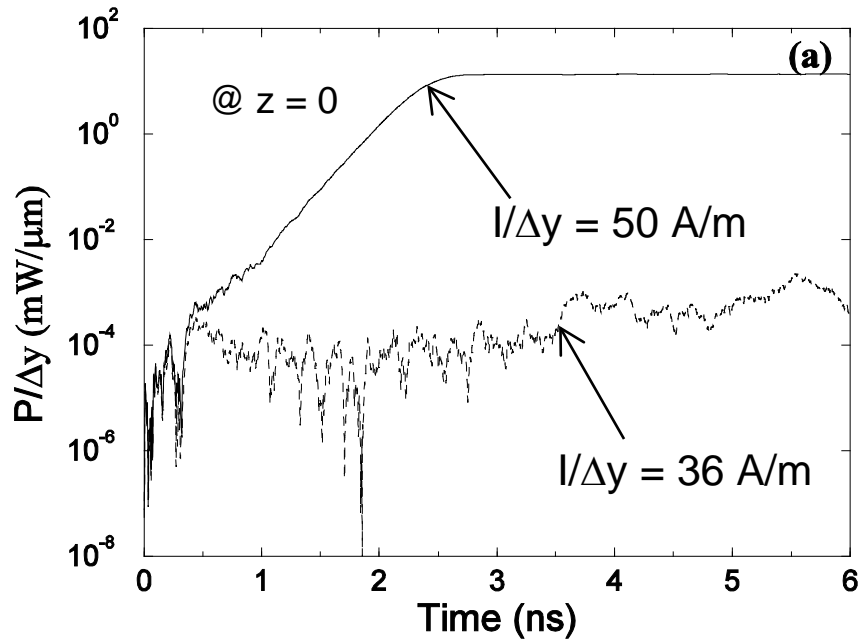
Numerical simulations



- Particles initialized as per Penman's algorithm to take care of shot noise
- Particle pushing for known field using predictor-corrector method.
- Convert P. D. E for field to difference equation and updated the field using the difference equation
- Grid parameters:
 $\Delta\tau = 0.01 \Rightarrow t = 2.02 \text{ ps}$
 $\Delta\zeta = 0.02, N_{\text{part}} = 1024$

*[Our algorithm is similar to N. S. Ginzburg et al., Sov. Radiophys. Electron., 21, 728 \(1979\).](#)

Simulation Results – I



For $I/\Delta y = 50 \text{ A/m}$, at saturation, $P/\Delta y = 13.7 \text{ mW}/\mu\text{m}$

Power e-folding time = 0.2 ns (simulation)

0.17 ns (analytic formula)

Lasing wavelength = 694.5 μm (simulation)

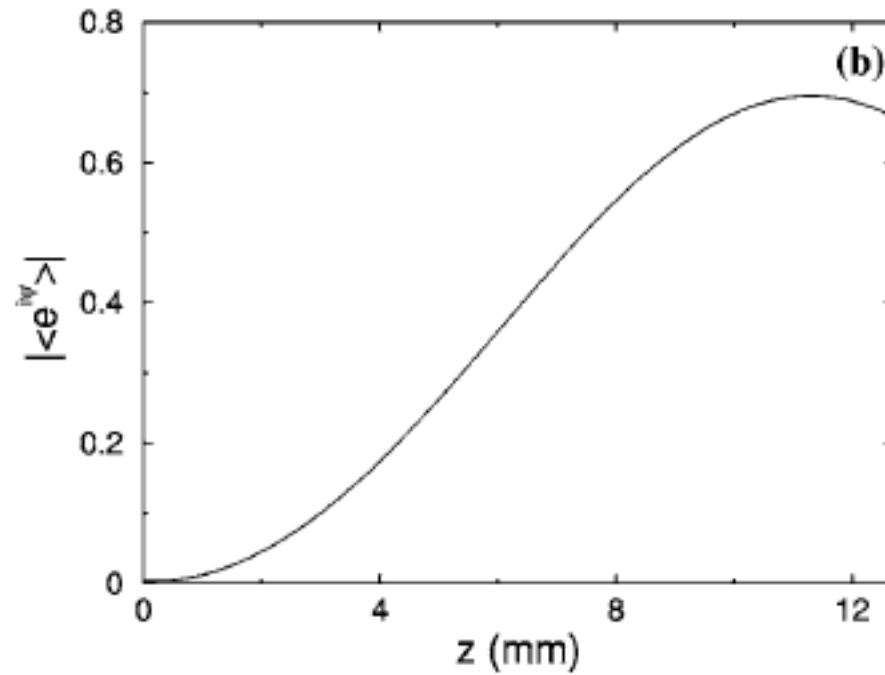
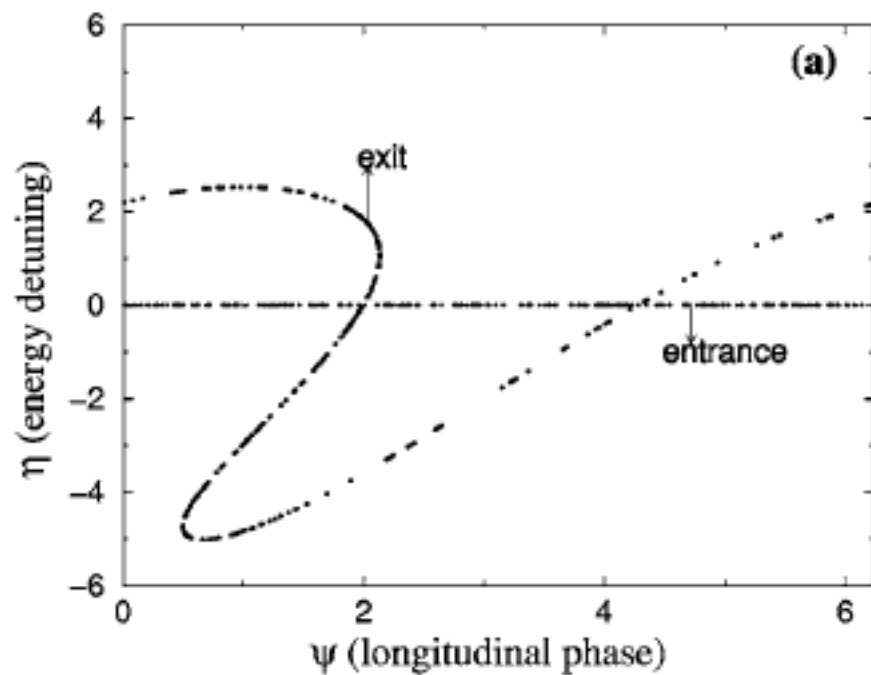
694 μm (analytic formula)

$$I_s = \mathcal{J}_s \frac{I_A \Delta y}{4\chi} \frac{\beta^4 \gamma^4}{kL^3} e^{2\Gamma_0 h}$$

$$\eta^{max} = \frac{\beta^3 \gamma^3}{(\gamma - 1)} \frac{\lambda}{L}$$

Energy conversion efficiency = 0.8%

Simulation Results – II



Bunching in Smith-Purcell FEL

Three Dimensional Effects

Three-dimensional surface mode can be constructed by combining the plane waves propagating at different angles with suitable weight factor¹.

$$\text{Beam sizes: } \Delta y = \sqrt{\lambda L / \pi \beta_g} \quad \Delta x = 1/2\Gamma_0$$

(~ few tens mm)

(~ few tens μm)

$$\text{Unnormalized emittance: } \epsilon_y \leq \frac{\lambda}{4\pi\beta_g} \quad \epsilon_x \leq \frac{(\Delta x)^2}{4L}$$

(~ few $\mu\text{m-rad}$)

(~ few nm-rad)

Gaussian mode in free space:

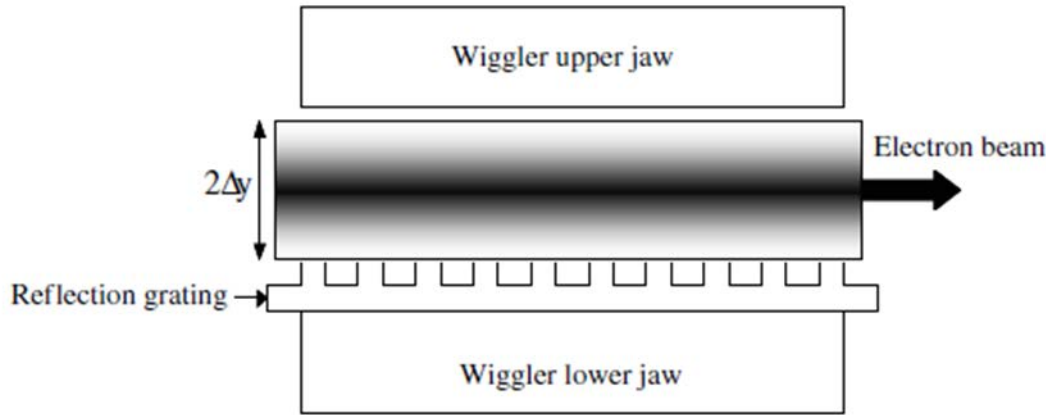
$$\text{RMS beam size} = \sqrt{\frac{\lambda Z_R}{4\pi}}$$

Need round to flat beam transformation to achieve this ².

The stringent criterion on vert. emitt. can be somewhat relaxed with the help of external focusing³.

1. **K.-J. Kim** and V. Kumar, PRSTAB **10**, 080702 (2007).
2. P. Piot, Y.-E. Sun, and **K.-J. Kim**, PRSTAB **09**, 031001 (2006).
3. V. Kumar and **K.-J. Kim**, PRSTAB **12**, 070703 (2009).

Relaxed beam parameters with external focusing¹



Grating parameters:

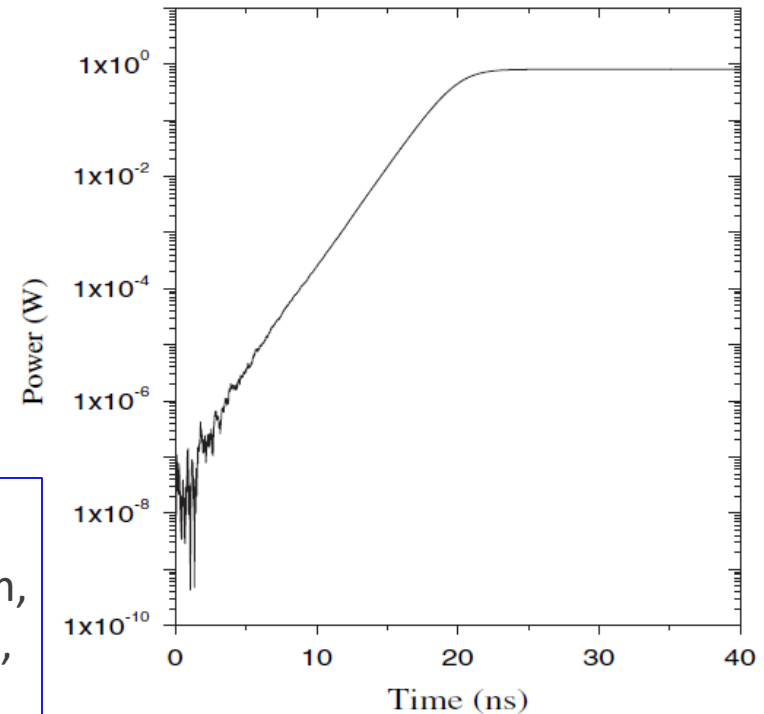
$$\begin{aligned}\lambda_g &= 173 \text{ } \mu\text{m}, \\ d &= 130 \text{ mm}, \\ w &= 110 \text{ } \mu\text{m}, \\ L &= 38.06 \text{ mm} \\ \lambda_R &= 761 \text{ mm}\end{aligned}$$

Beam parameters:

$$\begin{aligned}\beta &= 0.35 \text{ (35 keV)} \\ I &= 30 \text{ mA} \\ \Delta y &= 45.6 \text{ } \mu\text{m} \\ \Delta x &= 7 \text{ mm} \\ \beta\gamma\epsilon_x &= 10^{-5} \text{ mrad} \\ \beta\gamma\epsilon_y &= 10^{-7} \text{ mrad}\end{aligned}$$

Wiggler²:

$$\begin{aligned}\lambda_w &= 2.7 \text{ mm}, \\ \text{gap} &= 1 \text{ mm}, \\ B_{pk} &= 4.7 \text{ kG}\end{aligned}$$



Similar performance can be achieved also by using a solenoid (instead of wiggler) having a field of 2.5 kG to focus the electron beam, the flat beam in that case should be generated using a line cathode. The line cathode followed by a reflection grating should be immersed together in the uniform field region of the solenoid.

1. V. Kumar and **K.-J. Kim**, PRSTAB **12**, 070703 (2009).

2. V. A. Papadichev V. E. Rybalchenko, NIMA **19**, 407 (1998).

Conclusions

- Smith-Purcell FELs seem to be interesting and useful alternative to undulator based FELs for compact sources of terahertz radiation.
- SP-FEL can be described by Maxwell-Lorentz equations of the form similar to that of undulator based FELs. **Effect of attenuation and diffraction are particularly important, and can not be ignored.**
- Main challenge in achieving successful lasing in SP-FEL is to meet the stringent criteria on beam quality. Flat beam with low emittance in vertical direction is needed.
- If the challenge on beam quality is met in future, it appears that SP-FELs will be very useful as a compact source to generate average power of the order of Watt at terahertz wavelengths.



Observation of THz evanescent waves in a Smith-Purcell free-electron laser

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Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA

C.F. Guertin, A. O'Donnell, B. Durant, T.H. Lowell, and M.R. Mross

Vermont Photonics, Bellows Falls, Vermont 05101, USA

(Received 4 March 2009; published 19 August 2009)

We present experimental observations of evanescent waves in a Smith-Purcell free-electron laser (FEL). These waves, predicted by both theory and simulations, have wavelengths longer than the Smith-Purcell radiation, group velocity antiparallel to the electron beam, and for sufficiently high current, provide feedback to bunch the electron beam. This feedback is the basis of oscillator operation of the Smith-Purcell FEL. The wavelengths observed agree with theoretical predictions, and strong radiation from the upstream end of the grating confirms the negative group velocity. Radiation observed at the second harmonic may indicate electron bunching by the evanescent wave.

DOI: [10.1103/PhysRevSTAB.12.080703](https://doi.org/10.1103/PhysRevSTAB.12.080703)

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EMITTANCE MEASUREMENT OF A DC GUN FOR SMITH-PURCELL BACKWARD WAVE OSCILLATOR FEL*

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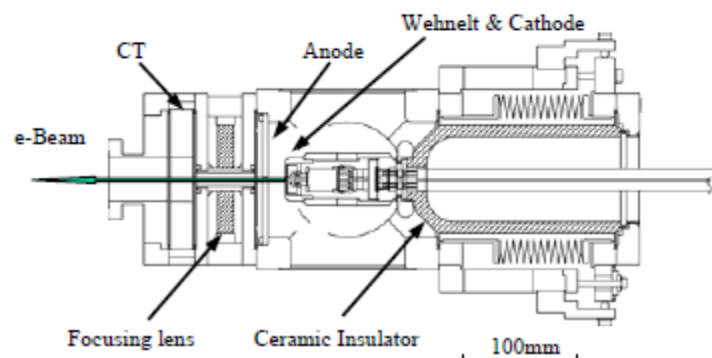
Abstract

A terahertz light source based on Smith-Purcell Backward Wave Oscillator FEL (S-P BWO-FEL) has been studied at Laboratory of Nuclear Science, Tohoku University. The DC gun employs a high voltage of 50 kV to extract electrons, which is suitable to drive S-P BWO-FEL [1]. Emittance measurement has been performed by means of a double slit technique. The deduced normalized rms emittance is about 2π mm mrad. We present the results of emittance measurement and analysis.

INTRODUCTION

Currently, various sources of terahertz (THz) radiation are based on accelerator or laser and semiconductor technology. Especially, the accelerator based sources have the potential to serve the THz light with very high intensity. It is expected that such THz radiation will be

power supply and cathode heater power supply with the feedback system is required to generate a high stability beam. The accelerating voltage was measured by a high voltage probe. It was found the fraction of the accelerating voltage is less than about $\pm 0.4\%$ during one measurement of charge intensity profile. In addition the cathode has been operated for 6000 hours without failure.



**Design and test of low emittance electron beam for
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